SECTION C — (3 × 10 = 30 marks)

Answer any THREE questions.

16. Prove that the outer measure of an interval equals its length.

17. State and Prove Fatou's lemma and give an example where strict inequality occurs.

18. State more precisely and prove every continuous function on a compact interval can be uniformly approximated by a polynomial.

19. Derive the criterion for determining when two mixed partials $D_{1,2}f$ and $D_{2,1}f$ will be equal.

20. State and prove implicit function theorem.

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APRIL/MAY 2024

23PMA22 — REAL ANALYSIS – II

Shanmuga Industries Arts & Science College T.V.Malai

Time : Three hours Maximum : 75 marks

SECTION A — (10 × 2 = 20 marks)

Answer ALL questions.

1. How can you define Lebesgue outer measure of a set?

2. If f is a measurable function and B is a Borel set, then prove that $f^{-1}(B)$ is measurable.

3. Show that if f and g are measurable and $|f| \leq |g|$ a.e., and g is integrable, then f is integrable.

4. State Lebesgue's dominated convergence theorem.

5. Define Dirichlet's kernel.

6. Compare Dini's test and Jordan's test for a convergence of a Fourier series.

7. What the matrix form of a chain rule.

8. Write the first order Taylor formula.

9. If $f = u + iv$ is a complex valued function with a derivative at a point 'z' in C, then prove that $J_f(z) = |f'(z)|^2$.

10. Define saddle point of a differentiable function.

SECTION B — (5 × 5 = 25 marks)

Answer ALL questions.

11. (a) Is the outer measure, translation invariant? Validate with suitable argument.

Or

(b) Derive Monotone convergence theorem with the use of Fatou's lemma

12. (a) If f is an integrable function, $\left|\int f dx\right| \leq \int |f| dx$, when does equality occur?

Or

(b) $f \in L(a+h, b+h)$ and $f_h(x) \equiv f(x+h)$, then show that $f_h \in L(a, b)$ and $\int_{a+h}^{b+h} f\,dx = \int_a^b f_h dx$.

13. (a) State and Prove Riesz-Fischer theorem.

Or

(b) Derive Riemann-Lebesgue lemma.

14. (a) Let $f = u + iv$ be a complex valued function on a complex variable. Show that the Cauchy-Riemann equations along with differentiability of 'u' and 'v' imply the existence of $f'(c)$ where 'c' is an interior point.

Or

(b) Let 'f' and 'g' be functions such that the composition $h = f \circ g$ defined in a neighbourhood of a point 'a'. How to compute the total derivative of 'h' in terms of total derivative of 'f' and 'g'.

15. (a) Derive the second derivative test for extrema.

Or

(b) A quadratic surface with centre at the origin has the equation

$Ax^2 + By^2 + Cz^2 + 2Dyz + 2Ezx + 2Fxy = 1$.

Find the lengths of its semi-axes.